Branched Eden clusters in the dynamic epidemic model

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The dynamic epidemic model [J. Phys. A **29**, 309 (1996)] considers a simple Eden growth history in a medium containing a fraction x of mobile hindrances (''particles''). These particles are supposed to be pushed by the front of the growing cluster. We have investigated how the medium becomes organized after n successive Eden growth cycles. Applications to various fields are underlined. Unexpectedly, the mobile hindrances aggregate such that the Eden cluster becomes branched after a finite number of growth cycles. [S1063-651X(98)05407-5]

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The formation of patterns reaching a high level of complexity is a subject of intense research in physics. Two famous paradigms of *out of equilibrium* structure development are certainly the Eden [1] and diffusion-limited aggregation (DLA) [2] growth models. They describe the growth of rough interfaces of compact structures and the formation of dendritic structures, respectively [3].

Recently, we have elaborated the so-called dynamic epidemic model [4] based on an Eden growth in order to study the growth of an interface perturbed by the presence of mobile hindrances. A repulsive interaction between the growth front and the so-called ''particles'' is assumed to occur. This model has been introduced [4,5] to describe the decoration of crystal by impurities. Other applications can be found in the trapping of bubbles during solidification [6], the growth of $YBa_2Cu_3O_{7-x}/Y_2BaCuO_5$ superconducting composites [7], a granular medium reached by a shock front or a traffic problem [8], the motion of flux lines near a diamagnetic domain like in high-T_c superconductors [9], or the behavior of biological cells on an ice-water front [10].

Initially, let each site (or cell) of a two-dimensional square lattice be occupied by a particle with a probability x or be empty with a probability $(1-x)$. The Eden-like growth starts by dropping a seed at the center of the lattice. At each growth step, all empty sites in contact with the growing cluster are selected. One of them is randomly chosen and is supposed to belong to the Eden cluster. If a particle is touched by the front, this particle makes a random move towards a nearest-neighbor site in order to reduce its contact with the Eden front (see Fig. 1). If the particle cannot reduce its number of nearest-neighboring cluster units by such a jump, the position of the particle remains unchanged and later becomes trapped by the front in the cluster. The site selection, cluster growth, and particle motion process described here is repeated a large number of times *if possible*. Indeed, the cluster growth can sometimes stop if there is no empty site in contact with the front as in static epidemic percolation [11]. The growth is irreversible (i.e., far from equilibrium) and history dependent (i.e., non-Markovian).

Previous investigations [4,5] have shown that the above dynamic epidemic model exhibits a blocking transition at $x_c = 0.56 \pm 0.01$. Below x_c, the clusters can grow forever while above x_c, the clusters stop growing. It should be pointed out that the critical value $x_c \approx 0.56$ is much higher than ≈ 0.41 for the model with static particles, i.e., the case of classical percolation [11,12]. Recently, it has been shown [13] that x_c can be mapped to the static percolation problem using the so-called Galam-Mauger formula [14]. Qualitatively, x_c increases with the coordination number z, i.e., the number of nearest neighbors of the lattice.

It has already been observed [5] that below x_c, *the growing front organizes the random medium*. More precisely, the pushed particles become aggregated and aggregates of particles are then engulfed by the growing cluster. The average displacements δ of particles are, however, finite [5]. The average displacement δ is a nontrivial function of x, i.e., not as simple as a power law, which vanishes when approaching the percolation blocking concentration x_c [5].

All studies we know only consider that the cluster stops growing or otherwise is infinite and contains an infinite percolation path [12]. Here, each growth is arbitrarily stopped when the cluster reaches a predetermined finite size N called the ''history time.''

The aim of the present paper is to show whether and how the medium becomes organized during and after several n successive Eden growths of identical time histories N and for $x < x_c$. Thus, in between successive Eden growths, the cluster is ''removed from'' the lattice but the dispersion of particles is left in the state reached at the end of the history.

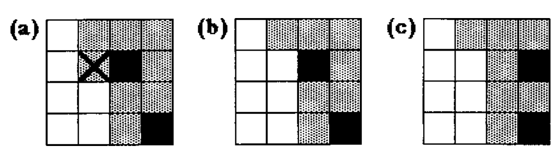

FIG. 1. Illustration of one growth step of the dynamic epidemic model. The growing phase is drawn in white, the mobile particles are drawn in black, and the empty sites are shown in gray. (a) One empty site denoted by a cross in contact with the growing cluster is selected at random; (b) this gives a new (white) unit on that site; (c) the mobile particle touched by the new unit jumps towards a nearest-neighboring site in order to reduce its contact with the cluster.

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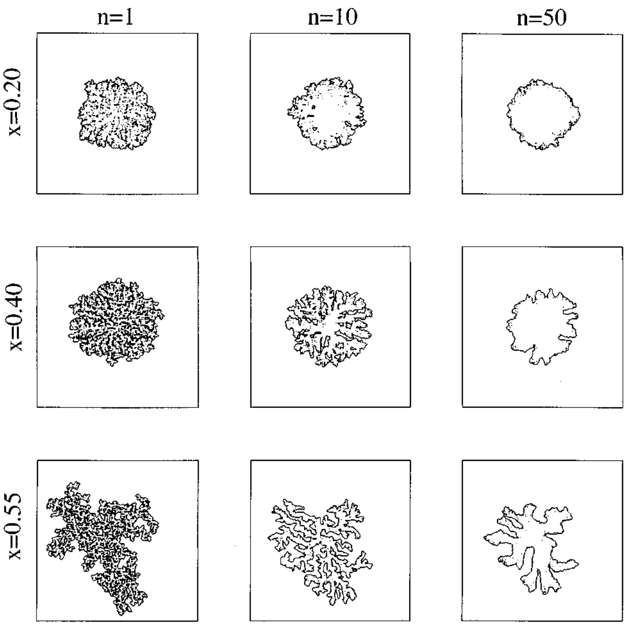

FIG. 2. Typical clusters of time history $N=5000$ obtained after $n=1$, 10, and 50 cycles. Cases with different particle concentrations are illustrated: $x=0.20$, 0.40, and 0.55.

Two specific examples serve as arguments. The present investigations are relevant for the water transport of salts in porous media like cementeous materials. Salts are left over by water in the pores during warming conditions. Capillarity forces driving the water invasion during wetting conditions greatly depend on the salt dispersion in pores [15]. In this case, cycles correspond to the porous materials being exposed to bad weather conditions. A second example is the displacements of magnetic flux lines in a superconductor. Flux lines can be pinned on defects. If the temperature further decreases, some diamagnetic domain nucleates and flux lines can be expelled along the boundaries of the diamagnetic cluster [9]. If the temperature increases, flux lines remain there since they are pinned. Thus, the present work is relevant for the case of an oscillating temperature near T_{c2}. The major ingredient to be studied herein is the memory effect of the medium configuration after each cycle indeed.

Figure 2 presents typical cluster perimeters for $N=5000$ obtained after $n=1$, 10, and 50 growths. Three different particle fractions x are illustrated: $x=0.20$, 0.40, and 0.55. After the first growth $(n=1)$, the clusters are round and compact as for simple Eden clusters. The black dots observed in the cluster are lacunes since particles (or even small aggregates) are trapped therein. The aggregated particles are seen to be trapped along radial filamentary substructures. This structure of trapped particles has been observed in fact in, e.g., the directional solidification of reinforced aluminum-based materials [16]. This internal patterning indicates that some organization of the random medium takes place as shown by the model. For $x\approx x_c$ (third row of Fig. 2), the clusters present holes, which are signatures of fractal patterns near the blocking transition. After $n=10$ cycles, the aggregates of particles seem to be expelled from the center of the growing cluster. Moreover, the accumulation of these hindrances leads to an unexpected result: *the cluster becomes branched*. This is well observed for $x\approx 0.40$. After $n=50$ cycles, the clusters recover a round and compact shape. However, this seems not to be the case for $x\approx x_c$. Indeed, branches are still seen in the $x=0.55$ picture.

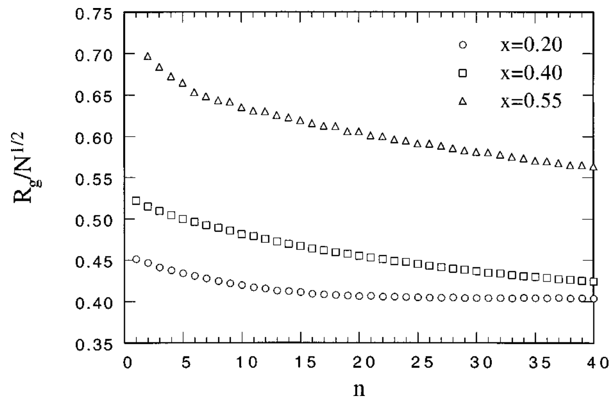

FIG. 3. Dependence of the gyration radius R_g on the number of successive growth cycles n. The cluster size is $N=5000$. Three particle fractions are illustrated: $x=0.20$, 0.40, and 0.55.

The evolution of the gyration radius R_g of $N=5000$ clusters as a function of the number n of cycles is shown in Fig. 3. Three different fractions of particles are illustrated: $x=0.20$, 0.40, and 0.55. The gyration radius seems to decrease exponentially from $[(1+\gamma x)N]^{1/2}$ for $n=1$ towards $N^{1/2}$ for $n\rightarrow\infty$. Since particles are trapped inside the clusters for low n values and are expelled out of the cluster when $n\rightarrow\infty$, the gyration radius is expected to have the following form:

$$R_g \sim \{[1+\gamma x(1-e_n)]N\}^{\nu}, \tag{1}$$

where e_n is the fraction of particles expelled out of the cluster and γ is a constant. The exponent ν in Eq. (1) is clearly $\nu=1/2$ and constant with n since the clusters remain dense (not fractal) for $x<x_c$.

The exponential decay of R_g as a function of n implies that the fraction e_n of particles repelled by the front reaches 1 exponentially. We propose to test the following behavior:

$$e_n = 1-\exp\left(-\frac{n\Delta}{N^{\alpha}}\right), \tag{2}$$

where $\Delta(x)$ is proportional to the average total displacement δ of particles.

A first observation to be made here concerns the above exponential decay itself, which implies the existence of a characteristic relaxation number

$$n^* \sim \frac{N^{\alpha}}{\Delta}. \tag{3}$$

This characteristic number n^* describes how the cluster relaxes towards a compact Eden cluster free of trapped particles. This number n^* diverges for $x\rightarrow x_c$ or for $N\rightarrow\infty$. Figure 4 presents a log-log plot of $1/n^*$ as a function of N. The exponent α has been numerically estimated to be $\alpha=0.70\pm 0.05$. This exponent α is significantly less than unity. This implies that the aggregation process occurring on the surface of the clusters is definitely a nonlinear process.

Notice that the exponential decay seen in Fig. 3 is somewhat unexpected. If one considers the finite displacements δ

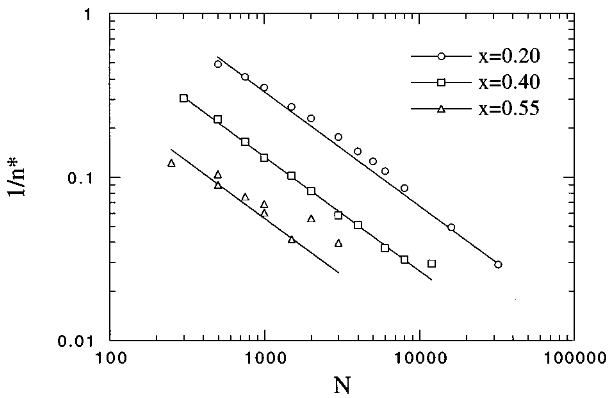

FIG. 4. Log-log plot of $1/n^*$ as a function of N. Three different particle fractions are illustrated: $x=0.20$, 0.40, and 0.55. The slope of the continuous line gives $\alpha=0.70\pm0.05$.

of particles and the conservation of the cluster shape as a function of n, it might be thought that the fraction of particles repelled should be proportional to the mean displacement δ of the particles and proportional to the surface of the cluster ($\sim R_g$) such that

$$xN\frac{de_n}{dn}\sim\delta\{[1+\gamma x(1-e_n)]N\}^{1/2} \tag{4}$$

rather giving a power-law decay for $(1-e_n)$ with an exponent 1/2. However, the hypothesis of round clusters *does not hold* with respect to the above numerical results. Thus, the exponential decay [Eq. (2)] implies the existence of some variation in the cluster morphology. As observed above, the variation of the shape is a branching effect in fact.

Computation time is quite long for $d=2$ lattices. Ivanova [17] considered recently $d=3$ cubic lattices for the dynamic epidemic lattice. It seems that trapped aggregates are distributed along filamentary patterns (like for $d=2$) oriented perpendicular to the surface. In so doing, we do not expect branched Eden clusters in $d=3$ since the two-dimensional surface is then pinned by aggregates.

In conclusion, the bridge between Eden and dendritic (such as DLA) growth pattern is captured by the present Eden-based model including memory effects. On the other hand, simple physical reasons have been introduced in order to show why the branching of Eden clusters is possible, and where the findings can be applied in various fields of scientific investigations.

Notice that extensions of the Eden model including probabilistic constraints on the growth leading to branches have already been reported as, for example, in [18]. The Eden tree model [19], which imposes the growth of a branched structure, has been also studied. However, it should be pointed out that the present model is not probabilistic and does not have particular rules *a priori* imposing the formation of branches: it results from a self-organization of the disordered medium in which the growth takes place.

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[1] R. Jullien and R. Botet, J. Phys. A **18**, 2279 (1985).
[2] H. E. Stanley, in *Fractals and Disordered Systems*, edited by A. Bunde and S. Havlin (Springer, Berlin, 1996), 2nd printing.
[3] *On Growth and Form*, edited by H. E. Stanley and N. Ostrowsky (Martinus Nijhoff, Dodrecht, 1986).
[4] N. Vandewalle and M. Ausloos, J. Phys. A **29**, 309 (1996).
[5] N. Vandewalle and M. Ausloos, Phys. Rev. Lett. **77**, 510 (1996).
[6] T. O'D. Hanley and T. Vo Van, J. Cryst. Growth **19**, 147 (1973).
[7] C.-J. Kim, S. H. Lai, and P. J. McGinn, Mater. Lett. **19**, 185 (1994).
[8] N. Pieret, honor thesis, University of Liège (1996).
[9] F. Bass, B. Ya. Shapiro, and M. Shvartser, Solid State Commun. **103**, 399 (1997).
[10] C. Körber, Q. Rev. Biophys. **21**, 229 (1988).
[11] A. Bunde, H. J. Herrmann, A. Margolina, and H. E. Stanley, Phys. Rev. Lett. **55**, 653 (1985).
[12] D. Stauffer and A. Aharony, *Introduction to Percolation Theory* (Taylor & Francis, London, 1992), 2nd printing.
[13] S. Galam *et al.* (unpublished).
[14] S. Galam and A. Mauger, Phys. Rev. E **53**, 2177 (1996); **56**, 322 (1997).
[15] D. M. Roy, invited paper at the MRS fall meeting 97, Boston, MA.
[16] U. Hecht and S. Rex, Metall. Mater. Trans. A **28**, 867 (1997).
[17] K. Ivanova, Phys. Rev. E **57**, 4827 (1998).
[18] G. Albinet and P. Pelce, Europhys. Lett. **33**, 569 (1996).
[19] D. Dhar, J. Phys. A **18**, L713 (1985).